

Time: 2 ½ Hrs

Marks: 75 Marks

N.B.

- 1) All questions are compulsory.
- 2) Figures to the right indicate marks.
- 3) Illustrations, in-depth answers and diagrams will be appreciated.
- 4) Mixing of sub-questions is not allowed.
- 5) Each question carries 5 Marks.

Q.1 Attempt any four of the following: 20 Marks

- a) Let the domain of $M = \{(x, y) : y = x\}$ be the set of real numbers. Is M a function?
- b) Sketch the following binary relations in $A \times A$, $A = \{\text{All real numbers}\}$.
 1. $R_1 : x^2 + y^2 = 4$
 2. $R_2 : x = 4y^2$
 3. $R_3 : (x^2 / 4) - y^2 = 1$
 Find if R_1 , R_2 , and R_3 functions?
- c) Solve the following:
 1. List all the ordered pairs in the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set $\{1, 2, 3, 4, 5, 6\}$.
 2. Display this relation graphically.
 3. Display this relation in tabular form.
- d) List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \in R$ if and only if
 1. $a = b$
 2. $a + b = 4$
 3. $a > b$
 4. $a \mid b$
 5. $\gcd(a, b) = 1$
- e) Solve the recurrence relation $F_n = 5F_{n-1} - 6F_{n-2}$ where $F_0 = 1$ and $F_1 = 4$.
- f) Computing its characteristic equation for The Fibonacci sequence $f_n = f_{n-1} + f_{n-2}$ is a homogeneous relation.

Q.2 Attempt any four of the following: 20 Marks

- a) How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards? Explain your answer.

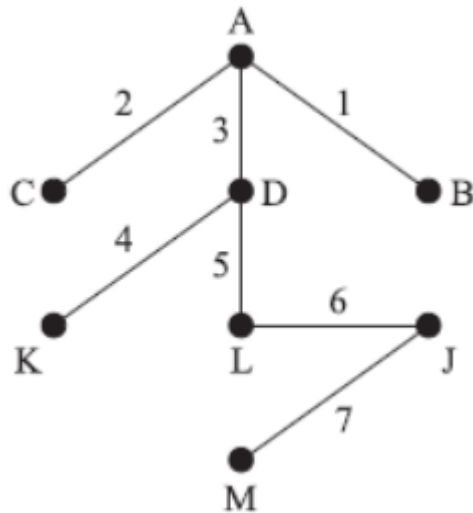
- b) There are 18 mathematics majors and 325 computer science majors at a college.
1. In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
 2. In how many ways can one representative be picked who is either a mathematics major or a computer science major?
- c) How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [Hint: First position the men and then consider possible positions for the women.]
- d) How many permutations of the letters ABCDEFGH contain
1. the string ED?
 2. the string CDE?
 3. the strings BA and FGH?
- e) One hundred tickets, numbered 1, 2, 3,..., 100, are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip to Tahiti). How many ways are there to award the prizes if,
1. there are no restrictions?
 2. the person holding ticket 47 wins the grand prize?
 3. the person holding ticket 47 wins one of the prizes?
- f) Construct a deterministic finite-state automaton that recognizes the set of all bit strings beginning with 01.

Q.3 Attempt any four of the following:

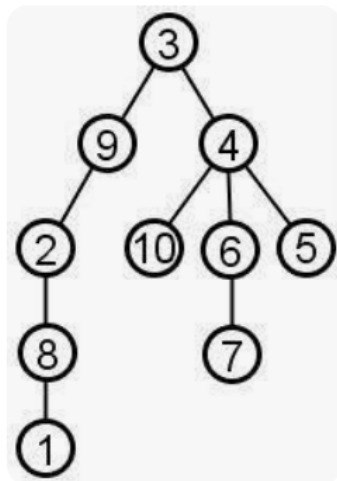
20 Marks

- a) Explain types of graphs.
- b) Let $P = \{1, 2, 3, 4\}$, $Q = \{a, b, c, d\}$ and $R = \{(1, a), (1, b), (1, c), (2, b), (2, c), (2, d)\}$. Find:
1. Adjacency Matrix
 2. Digraph
 3. Indegree
 4. Outdegree
 5. Type of graph

- c) Consider the graph G shown in the following image. Solve using BFS(Breadth-First search)algorithm step by step.



- d) Build a binary search tree for the words banana, peach, apple, pear, coconut, mango, and papaya using alphabetical order.
- e) For the given tree ,



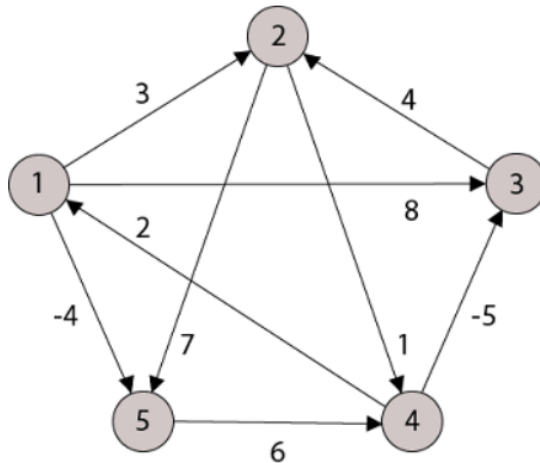
Find:

1. Leaves
 2. internal vertex
 3. Siblings of vertex 10
 4. ancestors of vertex 10
 5. Descendants of vertex 2
- f) Write an Algorithm for Inserting a vertex into a Binary Search Tree.

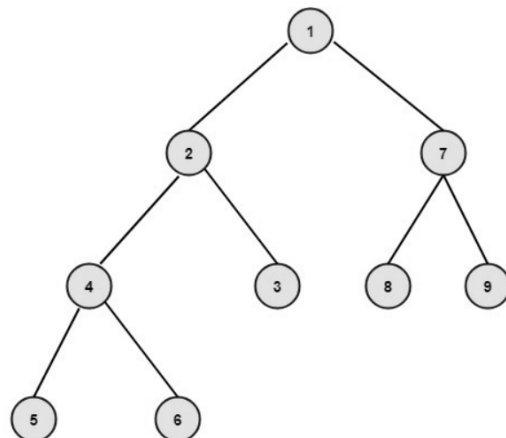
Q.4 Attempt any three of the following:

15 Marks

- a) Explain following terms:
1. Injective function
 2. Surjective function
 3. Bijective function
- b) Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = 2x$, is one-one and onto.
- c) For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.
1. $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
 2. $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
- d) What is a binary tree? Explain in detail.
- e) Apply Floyd-Warshall algorithm for constructing the shortest path. Show that matrices $D^{(k)}$ and $\pi^{(k)}$ computed by the Floyd-Warshall algorithm for the graph.



- f) Traverse the given tree T in preorder, postorder and Inorder:



***** END *****